Fractions: Of Amounts & Multiplying – Solutions

1. Fully simplify $\frac{18}{24}$: $\frac{18}{24} = \frac{3}{4}$

2. $2\frac{1}{3} + 1\frac{1}{2} = \frac{7}{3} + \frac{3}{2} = \frac{14}{6} + \frac{9}{6} = \frac{23}{6} = 3\frac{5}{6}$

3. Bar models:

$\frac{1}{2}$ of $20 = \mathbf{10}$ $\frac{1}{4}$ of $20 = \mathbf{5}$ $\frac{3}{4}$ of $20 = \mathbf{15}$

4. $\frac{1}{3}$ of $24 = 24 \div 3 = \mathbf{8}$ $\frac{2}{3}$ of $24 = 8 \times 2 = \mathbf{16}$

5. $\frac{3}{4}$ of $\square = 18$: 1 part $= 18 \div 3 = 6$, so whole $= 6 \times 4 = \mathbf{24}$

6. **True.** $\frac{1}{2}$ of 30 means splitting into 2 equal parts, which is the same as dividing by 2. So $\frac{1}{2}$ of $30 = 30 \div 2 = 15$. ✓

7. $\frac{2}{5}$ of 35: one fifth $= 35 \div 5 = 7$, so $\frac{2}{5} = 7 \times 2 = \mathbf{14}$

8. Amy's piece: $\frac{3}{4}$ of 60 cm $= (60 \div 4) \times 3 = 15 \times 3 = \mathbf{45\,cm}$

1. Order smallest to largest: Common denominator 24: $\frac{3}{8} = \frac{9}{24}$, $\frac{1}{2} = \frac{12}{24}$, $\frac{5}{12} = \frac{10}{24}$

$$\frac{3}{8} < \frac{5}{12} < \frac{1}{2}$$

2. $\frac{17}{5} = 3\frac{2}{5}$

3. Bar models: $\frac{3}{4}$ of 48: one quarter $= 12$, so $\frac{3}{4} = \mathbf{36}$ $\frac{5}{8}$ of 48: one eighth $= 6$, so $\frac{5}{8} = \mathbf{30}$

4. $\frac{5}{8}$ of 72: one eighth $= 72 \div 8 = 9$, so $\frac{5}{8} = 9 \times 5 = \mathbf{45}$

5. $\frac{2}{3}$ of 30 $= 20$ girls. Boys $= 30 - 20 = \mathbf{10}$

6. Rania: $\frac{3}{5}$ of $40 = (40 \div 5) \times 3 = 8 \times 3 = 24$. **Correct.** ✓

7. $\frac{3}{4}$ of $60 = 45$ vs $\frac{4}{5}$ of $55 = 44$. $\frac{3}{4}$ **of** 60 **is greater.**

8. $\frac{\square}{8}$ of $64 = 40$: one eighth of $64 = 8$, so $40 \div 8 = 5$. $\square = \mathbf{5}$

1. $\frac{1}{2}$ of $\frac{1}{2}$: Halving a half gives one quarter. $= \frac{1}{4}$

   (Picture: a square split into 4 equal parts; 1 part shaded.)

2. Area model (1 cell shaded out of 6 total):

$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

3. Area model (6 cells shaded out of 12 total):

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$$

4. $\frac{3}{5} \times \frac{5}{7} = \frac{15}{35} = \frac{3}{7}$

5. Simplify first: $\frac{3}{9} = \frac{1}{3}$, $\frac{4}{8} = \frac{1}{2}$. Then: $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

6. $\frac{3}{4}$ of $\frac{2}{5} = \frac{3}{4} \times \frac{2}{5} = \frac{6}{20} = \frac{3}{10}$

   Notice: "of" and "$\times$" give the same result – multiplying fractions is the same as finding a fraction of another fraction.

7. $1\frac{1}{2} \times \frac{1}{3}$: Convert: $\frac{3}{2} \times \frac{1}{3} = \frac{3}{6} = \frac{1}{2}$

1. $\dfrac{24}{36} = \dfrac{2}{3}$

2. $3\dfrac{1}{4} - 1\dfrac{2}{3}$:

$$\frac{13}{4} - \frac{5}{3} = \frac{39}{12} - \frac{20}{12} = \frac{19}{12} = 1\frac{7}{12}$$

3. $\dfrac{5}{6}$ of 42: $42 \div 6 = 7$; $7 \times 5 = \mathbf{35}$

4. Mural length: $\dfrac{2}{3}$ of $4\frac{1}{2}$ m

   Bar model: 3 equal parts each of length 1.5 m; mural covers 2 parts.

$$\frac{2}{3} \times \frac{9}{2} = \frac{18}{6} = 3\,\text{m}$$

   The mural is **3** m long.

5. Area model ($3 \times 4 = 12$ shaded out of $5 \times 7 = 35$ cells):

$$\frac{3}{5} \times \frac{4}{7} = \frac{12}{35}$$

6. $\dfrac{2}{3} \times \dfrac{3}{4} = \dfrac{6}{12} = \dfrac{1}{2}$ and $\dfrac{3}{4}$ of $\dfrac{2}{3} = \dfrac{3}{4} \times \dfrac{2}{3} = \dfrac{6}{12} = \dfrac{1}{2}$

   They are equal – multiplication of fractions is commutative.

7. $\dfrac{3}{5} \times \square = \dfrac{6}{35}$: $\square = \dfrac{6}{35} \div \dfrac{3}{5} = \dfrac{6}{35} \times \dfrac{5}{3} = \dfrac{30}{105} = \dfrac{2}{7}$

8. **True.** When both fractions are proper (between 0 and 1), each acts as a scaling factor less than 1. So the product is smaller than either

1. $\dfrac{20}{30} = \dfrac{2}{3}$

2. $\dfrac{84}{108}$: $\gcd(84, 108) = 12$; $= \dfrac{7}{9}$

3. $4\dfrac{3}{5} = \dfrac{4 \times 5 + 3}{5} = \dfrac{23}{5}$

4. $2\dfrac{3}{4} + 1\dfrac{3}{5} = \dfrac{11}{4} + \dfrac{8}{5} = \dfrac{55}{20} + \dfrac{32}{20} = \dfrac{87}{20} = 4\dfrac{7}{20}$

5. Bar model: 3 parts $= 27$, so 1 part $= 9$. Whole $= 4 \times 9 = \mathbf{36}$

6. Jack spends $\dfrac{1}{3} + \dfrac{1}{4} = \dfrac{4}{12} + \dfrac{3}{12} = \dfrac{7}{12}$.

   Remaining $= 1 - \dfrac{7}{12} = \dfrac{\mathbf{5}}{\mathbf{12}}$

7. $\dfrac{2}{3}$ of $1\dfrac{1}{2}\,\ell = \dfrac{2}{3} \times \dfrac{3}{2} = \dfrac{6}{6} = 1\,\ell = \mathbf{1000}\,\text{ml}$

8. Draw two area diagrams:

   Diagram A: Rectangle split into 3 columns, 4 rows; shade 2 cols $\times$ 3 rows $= 6$ cells out of 12. $\dfrac{2}{3} \times \dfrac{3}{4} = \dfrac{6}{12} = \dfrac{1}{2}$

   Diagram B: Same rectangle split into 4 columns, 3 rows; shade 3 cols $\times$ 2 rows $= 6$ cells out of 12. $\dfrac{3}{4} \times \dfrac{2}{3} = \dfrac{6}{12} = \dfrac{1}{2}$

   Both give the same shaded region $\Rightarrow$ multiplication is commutative.