Bearings Challenge

Bob knows the location of some buried gold, it lies exactly 100km away, on a line exactly 123.5° clockwise from North.

Bob has an electronic compass that can direct him on bearings, with no error, unfortunately, it will only accept 3 digit bearings, with no fractions of degrees.

To find the gold, Bob must arrive within 1m of its location.

1 Introduction

Assume all travellers can perfectly measure their distance travelled.

1. Using a scale of 1cm : 20km, sketch a map for Bob, label the angles and distances
2. Determine how Bob can locate the gold by travelling on a bearing of 180° followed by 090°
3. Bob can travel 20km a day, how long will it take him to travel to the gold?
4. Rob has stumbled upon the map to the gold, and intends to plot a course to arrive before Bob, travelling Southeast, then East. Armed with the same model of electronic compass, and travelling at the same speed, how much earlier will he arrive?
5. You want to beat both Bob and Rob to the treasure, but can also only follow 3 digit bearings. And now they both have a 7 hour head start! Covering 20km per day can you find a way to arrive at the gold first? is your route the quickest possible?

2 Challenge I

Previously we assumed that the travellers could measure their distances with unlimited precision - in reality that is not the case. Luckily, you, Bob and Rob have downloaded an app that will ping every time you travel exactly 1km in a straight line.

Therefore - all sections of journeys must be exact multiples of 1km!

1. Determine how far off the gold's location you, Bob and Rob will be if you round each leg of your planned journeys to the nearest kilometre. Thus show that the gold will remain buried.

2. What is the outcome of travelling the following route:

   - 2km on a bearing 179°
   - 2km on a bearing of 000°
   - 1km on a bearing of 182°
   - 1km on a bearing of 000°

3. What is the outcome of travelling the following route:

   - 2km on a bearing 091°
   - 2km on a bearing of 270°
   - 1km on a bearing of 088°
   - 1km on a bearing of 270°

4. Use the previous two questions to show that a route leading to the gold is possible - how long will this route take?

5. Show that it is possible to plot a route to within 1m of any location (x, y)

6. Show that it is possible to plot a route with at most 6 legs to within 1m of any location (x, y)

3 Challenge II

Now we restrict our attention to routes that have only two legs, thus take the following form:

- Travel Mkm on a bearing of $ABC°$
- Travel Nkm on a bearing of $XYZ°$

Where $000° \leq ABC, XYZ < 360°$, and $M, N \in \mathbb{N}$

1. Consider the case that $M > 6000$, what restriction does this place on $XYZ°$ to return within 100km of the start location?

2. Use the cosine rule to determine the range of N values possible given M, to ensure that the end of the journey lies between 0 and 100km from the start

3. Thus establish an upper bound on the number of possible two leg journeys that we need to consider

4. Does the conclusion from the previous section still necessarily hold?

5. Use the following table to refine the upper bound of the number of possible two leg journeys to consider:

	$ABC°$ Option count	M in range	$XYZ°$ Option count per M value (upper bound)	N Option count per M value (upper bound)	Total journey option count for this M range
$M \leq 100$					
$100 < M \leq 500$					
$500 < M \leq 2000$					
$2000 < M \leq 6000$					

6. Consider the journey:

   - Travel Nkm on a bearing of $XYZ°$
   - Travel Mkm on a bearing of $ABC°$

   and thus establish and upper bound U, on the number of two leg journeys that finish within 100km of the start point, with $U < 10^{10}$

7. Consider the area in square meters of a circle radius 100km, thus show that there are some locations that cannot be reached within 1m, following a two leg journey

8. Use a computer to determine if the gold can be reached via a two leg journey

9. If the gold can be reached, use a computer to find the closest point to the gold that cannot be reached; if the gold cannot be reached, find the point closest to the gold that can