Worked Solutions: Conversions and Basis Points

Question 1

Write the following decimals as a fraction in simplest form and as a percentage.

(a) 0.125

Solution:

$$0.125 = \frac{125}{1000}$$
$$= \frac{125 \div 125}{1000 \div 125}$$
$$= \frac{1}{8}$$

To convert to percentage, multiply by 100:

$$0.125 \times 100\% = 12.5\%$$

So, $0.125 = \frac{1}{8} = 12.5\%$.

(b) 0.203125

Solution:

$$0.203125 = \frac{203125}{1000000}$$
$$= \frac{203125 \div 15625}{1000000 \div 15625}$$
$$= \frac{13}{64}$$

Note: $15625 = 5^6$, and $203125 \div 15625 = 13$.

To convert to percentage:

$$0.203125 \times 100\% = 20.3125\%$$

So, $0.203125 = \frac{13}{64} = 20.3125\%$.

Question 2

Convert the following fractions into decimals and percentages.

  (a) $\frac{7}{20}$

    Solution:

$$\frac{7}{20} = 7 \div 20 = 0.35$$
$$0.35 \times 100\% = 35\%$$

    So, $\frac{7}{20} = 0.35 = 35\%$.

  (b) $\frac{33}{320}$

    Solution:

$$\frac{33}{320} = 33 \div 320 = 0.103125$$
$$0.103125 \times 100\% = 10.3125\%$$

    So, $\frac{33}{320} = 0.103125 = 10.3125\%$.

Question 3

Express the following percentages as decimals and fractions in simplest form.

  (a) 17.5%

    Solution:

$$17.5\% = \frac{17.5}{100} = 0.175$$
$$0.175 = \frac{175}{1000} = \frac{7}{40}$$

    So, $17.5\% = 0.175 = \frac{7}{40}$.

  (b) 57.8125%

    Solution:

$$57.8125\% = \frac{57.8125}{100} = 0.578125$$
$$0.578125 = \frac{578125}{1000000} = \frac{37}{64}$$

    Note: $578125 \div 15625 = 37$ and $1000000 \div 15625 = 64$.

    So, $57.8125\% = 0.578125 = \frac{37}{64}$.

Question 4

Basis points are units used in finance: 1 basis point $= 0.01\%$.

(a) Write one basis point as a decimal and a fraction in simplest form.

Solution:

$$1 \text{ basis point} = 0.01\%$$
$$= \frac{0.01}{100} = 0.0001$$
$$= \frac{1}{10000}$$

So, 1 basis point $= 0.0001 = \frac{1}{10000}$.

(b) Write 175 basis points as a decimal and a fraction in simplest form.

Solution:

$$175 \text{ basis points} = 175 \times 0.01\% = 1.75\%$$
$$= \frac{1.75}{100} = 0.0175$$
$$= \frac{175}{10000} = \frac{7}{400}$$

So, 175 basis points $= 0.0175 = \frac{7}{400}$.

Gilts are UK government bonds with a coupon payment equal to a percentage of the amount borrowed.

(c) Find 5.355% as a decimal and a fraction in its simplest form.

Solution:

$$5.355\% = \frac{5.355}{100} = 0.05355$$
$$= \frac{5355}{100000} = \frac{1071}{20000}$$

So, $5.355\% = 0.05355 = \frac{1071}{20000}$.

(d) Calculate the total cost for the government to borrow £1000 over 30 years, including payback of the original amount.

Solution:

$$\text{Annual coupon} = 5.355\% \text{ of } 1000$$
$$= 0.05355 \times 1000 = 53.55$$
$$\text{Total coupon payments over 30 years} = 30 \times 53.55 = 1606.50$$
$$\text{Repayment of principal} = 1000$$
$$\text{Total cost} = 1606.50 + 1000 = 2606.50$$

So, the total cost is £2606.50.

(e) Suppose the coupon payment of a 30 year gilt rises 43.5 basis points. How much more does it cost the government to borrow £1000 over 30 years?

Solution:

$$
\begin{aligned}
43.5 \text{ basis points} &= 43.5 \times 0.01\% = 0.435\% \\
\text{Increase in annual coupon} &= 0.435\% \text{ of } 1000 \\
&= 0.00435 \times 1000 = 4.35 \\
\text{Additional cost over 30 years} &= 30 \times 4.35 = 130.50
\end{aligned}
$$

So, it costs an additional £130.50.