

Iterative Processes

Iterative Processes • Tables of Values • n th Terms

Name: Class: Date:

Key Ideas

An iterative process links each term to the one before it.

Example: $U_{n+1} = U_n + 3, U_1 = 5$

This means: *add 3 to the previous term, starting at 5.*

Terms: 5, 8, 11, 14, ...

An n th term formula gives any term directly.

Example: $U_n = 3n + 2$

This means: *substitute any n to get that term immediately.*

Section A Iterative Process \rightarrow Table of Values

A1. Consider the iterative process

$$U_{n+1} = U_n + 7, \quad U_1 = 4$$

Getting Started: To find U_2 , substitute $n = 1$: $U_2 = U_1 + 7 = 4 + 7 = 11$. Now continue in the same way for each subsequent term.

(a) Complete the table of values below.

n	1	2	3	4	5	6
U_n	4					

(b) Describe in words what this sequence is doing.

A2. Consider the iterative process

$$V_{k+1} = 3V_k, \quad V_1 = 2$$

Getting Started: The operation here is multiplication, not addition. To find V_2 : $V_2 = 3 \times V_1 = 3 \times 2 = 6$. Notice the variable is V_k , not U_n — the process works in exactly the same way.

(a) Complete the table of values below.

k	1	2	3	4	5
V_k	2				

(b) Describe in word what the sequence is doing.

A3. Consider the iterative process

$$F_{j+1} = F_j - 9, \quad F_0 = 72$$

Getting Started: This sequence starts at $j = 0$, not $j = 1$. The first entry in your table is therefore $F_0 = 72$. Find $F_1 = F_0 - 9 = 72 - 9 = 63$, and continue from there.

(a) Complete the table of values below.

j	0	1	2	3	4	5	6
F_j	72						

(b) For what value of j does F_j first become negative? Show how you know.

A4. Consider the iterative process

$$P_{n+1} = \frac{P_n}{4}, \quad P_1 = 1024$$

Getting Started: Dividing by 4 is the same as multiplying by $\frac{1}{4}$. Start: $P_1 = 1024$, so $P_2 = \frac{1024}{4} = 256$. Continue until you have 5 terms.

(a) Complete the table of values below.

n	1	2	3	4	5
P_n	1024				

(b) Write each value of P_n as a power of 4. The first two are done for you.

n	1	2	3	4	5
P_n	4^5	4^4			

(c) Use the pattern to write an n th term formula for P_n directly.

$$P_n =$$

(d) Write P_n in the form $P_n = \text{---} \times \left(\frac{1}{4}\right)^n$:

$$P_n =$$

Section B Different representations of sequences

Each row below shows the **same sequence** in three different forms. One cell in each row is already filled in — complete the rest.

Getting Started: You already know some methods to find the n th term of sequences, however if the sequence uses multiplication then consider your answer to **A4**, and previous work on compounding growth. Use substitution to check your answers. Be careful, some answers will use a different variable to n in the “ n th” term!

B1.

Iterative Process	Table (first 5 terms)	n th Term Formula
	5, 8, 11, 14, 17...	

B2.

Iterative Process	Table (first 5 terms)	n th Term Formula
$F_{j+1} = \frac{1}{2}F_j, \quad F_1 = 80$		

B3. A quantity decreases by 15% each step, starting at $V_0 = 200$.

Iterative Process	Table ($n = 0$ to 4, 2 d.p.)	n th Term Formula

B4.

Iterative Process	Table (first 5 terms, 2 d.p.)	n th Term Formula
$Q_{n+1} = \frac{Q_n}{2} + 6, \quad Q_1 = 100$		<p><i>No neat formula exists — see question below</i></p>

What is different about this question?

Section C Context Problems — When Does the Sequence Start?

In these questions you must decide whether the sequence should start at $n = 0$ or $n = 1$, and justify your choice.

C1 — Savings Account

Amara opens a savings account with £600. Each year, 5% interest is added at the end of the year. Let V_k be the value of the account **after** k **complete years**.

Getting Started: Interest is added at the end of each year. Before any interest has been added (at year 0), the account holds the original £600. So what is V_0 ? What does V_1 represent?

- (a) State the value of V_0 and explain why the sequence starts at $k = 0$ rather than $k = 1$.

- (b) Write an iterative process for V_{k+1} in terms of V_k .

$$V_{k+1} = \quad , \quad V_0 =$$

- (c) Complete the table for $k = 0$ to 4. Give values to the nearest penny.

k	0	1	2	3	4
V_k (£)	£600.00				

- (d) Write a formula for V_k directly. Use it to calculate V_{10} (to the nearest penny).

$$V_k =$$

$$V_{10} =$$

C2 — Bouncing Ball

A ball is dropped from a height of 4 m. Each bounce reaches 75% of the previous height.

Jess defines H_n as the height reached after n bounces ignoring the initial drop.

Kwame defines H_n as the height after n bounces, where the *original drop height counts as the height after 0 bounces*.

- (a) Write Jess's iterative process and state her starting value.

$$H_{n+1} = \quad , \quad H_1 =$$

- (b) Write Kwame's iterative process and state his starting value.

$$H_{n+1} = \quad , \quad H_0 =$$

- (c) What is the same about Jess's and Kwame's iterative processes?

- (d) What is different about Jess's and Kwame's iterative processes?

- (e) For **each** definition, state what height H_3 represents, and calculate its value.

Jess: $H_3 =$ _____ m. This represents: _____

Kwame: $H_3 =$ _____ m. This represents: _____

- (f) Whose definition do you prefer?

Section D Extension and Challenge Problems

D1 — Working Backwards

The 5th term of an iterative sequence is $U_5 = 48$.

The iterative process is $U_{n+1} = 2U_n$.

- (a) Find U_1 . Show all steps clearly.

- (b) Write an n th term formula for U_n . Use it to verify that $U_5 = 48$.

D2 — Convergence

Consider the iterative process $F_{j+1} = \frac{F_j}{2} + 6$, $F_1 = 100$

(a) Complete the table to 2 decimal places.

j	1	2	3	4	5	6	7	8
F_j	100							

(b) What value does F_j appear to be approaching?

(c) If the sequence eventually settled at a fixed value L (called a **fixed point**), it would satisfy

$$L = \frac{L}{2} + 6$$

Solve this equation to find L .

(d) What do you notice about your answers to (b) and (c)? Do you think F_j will ever have a term that has the same value as L ?
