Worked Solutions: Ratio Extension Problems

Simplifying Ratios Past

1. In a survey, 60% of people preferred tea and 40% preferred coffee. Write the ratio of tea drinkers to coffee drinkers in its simplest form.

   Solution:
   $$\text{Tea} : \text{Coffee} = 60\% : 40\% = 60 : 40 = \frac{60}{20} : \frac{40}{20} = 3 : 2$$

2. A shirt is on sale for 30% off. The discount amount is £12. What is the ratio of the discount amount to the original price? Simplify your answer.

   Solution:
   Let the original price be P. Discount = 30% of P = £12.
   $$0.3P = 12 \quad \Rightarrow \quad P = \frac{12}{0.3} = 40$$
   Original price = £40. Discount = £12.
   $$\text{Ratio} = 12 : 40 = \frac{12}{4} : \frac{40}{4} = 3 : 10$$

3. A battery has lost 15% of its charge. What is the ratio of the charge lost to the charge remaining? Simplify your answer.

   Solution:
   Let total charge = 100%. Charge lost = 15%. Charge remaining = 85%.
   $$\text{Lost} : \text{Remaining} = 15 : 85 = \frac{15}{5} : \frac{85}{5} = 3 : 17$$

4. In a year, a company's profits were allocated as follows: 45% was reinvested, 20% was paid as dividends, and the rest was used for employee bonuses.

   (a) What percentage was used for employee bonuses?
   Solution:
   $$100\% - (45\% + 20\%) = 100\% - 65\% = 35\%$$

   (b) Express the ratio Reinvestment : Dividends : Bonuses in its simplest form.
   Solution:
   $$45 : 20 : 35 = \frac{45}{5} : \frac{20}{5} : \frac{35}{5} = 9 : 4 : 7$$

5. The population of a town is 12,500. After a census, it's found that the population has increased by 18%. What is the ratio of the new population to the original population? Express your answer in the form $a : b$, where a and b are integers.

   Solution:
   Increase = 18% of 12,500 = $0.18 \times 12500 = 2250$.
   New population = $12500 + 2250 = 14750$.

$$\text{Ratio} = 14750 : 12500 = \frac{14750}{250} : \frac{12500}{250} = 59 : 50$$

Simplifying Ratios Future

6. The two rectangles below are similar. Find the ratio of the perimeter of rectangle A to the perimeter of rectangle B. Then, find the ratio of the area of rectangle A to the area of rectangle B.

   Solution:
   Rectangle A: 2 cm × 1 cm. Rectangle B: 4 cm × 2 cm.
   Scale factor from A to B: $\frac{4}{2} = 2$ (or $\frac{2}{1} = 2$).
   Perimeter ratio = scale factor = $2 : 1$.
   Area ratio = $(\text{scale factor})^2 = 2^2 : 1^2 = 4 : 1$.

7. A square has a side length of s. Another square has a side length of $3s$. What is the ratio of the perimeter of the smaller square to the larger square? What is the ratio of their areas?

   Solution:
   Perimeter smaller = $4s$, perimeter larger = $4(3s) = 12s$.
   Perimeter ratio = $4s : 12s = 1 : 3$.
   Area smaller = s^2, area larger = $(3s)^2 = 9s^2$.
   Area ratio = $s^2 : 9s^2 = 1 : 9$.

8. A right-angled triangle has a base of 8 cm and a height of 6 cm. A similar triangle has a base of 12 cm.

   (a) What is the scale factor from the smaller to the larger triangle?
   Solution:

$$\text{Scale factor} = \frac{12}{8} = 1.5 = \frac{3}{2}$$

   (b) What is the ratio of the perimeter of the smaller triangle to the larger triangle?
   Solution:
   Perimeter ratio = $1 : \frac{3}{2} = 2 : 3$.

   (c) What is the ratio of the area of the smaller triangle to the larger triangle?
   Solution:
   Area ratio = $1^2 : \left(\frac{3}{2}\right)^2 = 1 : \frac{9}{4} = 4 : 9$.

9. The diagram below shows a large rectangle with a smaller rectangle cut out of it. Find the ratio of the area of the shaded region to the area of the unshaded (cut-out) rectangle.

Solution:
Large rectangle area $= 8 \times 6 = 48$ m^2.
Unshaded rectangle area $= 4 \times 2 = 8$ m^2.
Shaded area $= 48 - 8 = 40$ m^2.

$$\text{Ratio} = 40 : 8 = 5 : 1$$

10. A circular pond has a radius of r. A path is built around it, creating a larger circle with a radius of R, where $R = 2r$.

    (a) Find the ratio of the circumference of the pond to the circumference of the outer edge of the path.

    Solution:
    Circumference pond $= 2\pi r$, circumference outer edge $= 2\pi R = 2\pi(2r) = 4\pi r$.
    Ratio $= 2\pi r : 4\pi r = 1 : 2$.

    (b) Find the ratio of the area of the pond to the total area enclosed by the outer edge of the path.

    Solution:
    Area pond $= \pi r^2$, total area $= \pi R^2 = \pi(2r)^2 = 4\pi r^2$.
    Ratio $= \pi r^2 : 4\pi r^2 = 1 : 4$.

Sharing Into Ratios Past

11. A sum of £80 is shared between two people. Person A receives 60% of the total. What is the ratio of their shares, and how much does each person receive?

    Solution:
    Person A: 60%, Person B: 40%. Ratio $= 60 : 40 = 3 : 2$.
    Amount A $= 0.6 \times 80 = 48$.
    Amount B $= 80 - 48 = 32$.

12. In a class, 40% of students are boys and the rest are girls. The school fund of £150 is to be shared between boys and girls in the same ratio as their numbers in the class. How much do the boys get?

    Solution:
    Boys: 40%, Girls: 60%. Ratio $= 40 : 60 = 2 : 3$.
    Total parts $= 2 + 3 = 5$.
    Boys' share $= \frac{2}{5} \times 150 = 60$.

13. A bonus of £2,500 is shared between three departments in proportion to their percentage contribution to company profits. Department A contributed 25%, Department B 35%, and Department C the remainder. How much does each department receive, and what is the ratio of their shares in simplest form?

    Solution:
    Department C $= 100\% - (25\% + 35\%) = 40\%$.
    Ratio $= 25 : 35 : 40 = 5 : 7 : 8$ (dividing by 5).
    Total parts $= 5 + 7 + 8 = 20$.
    Amount A $= \frac{5}{20} \times 2500 = 625$.

Amount B = $\frac{7}{20} \times 2500 = 875$.
Amount C = $\frac{8}{20} \times 2500 = 1000$.

14. In a recipe, the dry ingredients (flour and sugar) are mixed in the ratio 3:2 by weight. If the flour makes up 30% of the total mixture (including 400g of wet ingredients), find the weight of flour and sugar used.

    Solution:
    Let total mixture weight = T. Flour = 30% of T.
    Dry ingredients = flour + sugar. Flour : sugar = 3:2, so flour = $\frac{3}{5}$ of dry weight.
    But flour = 0.3T, so dry weight = $\frac{5}{3} \times 0.3T = 0.5T$.
    Wet ingredients = $T - 0.5T = 0.5T = 400$ g.
    Thus $T = 800$ g.
    Flour = $0.3 \times 800 = 240$ g.
    Dry weight = $0.5 \times 800 = 400$ g.
    Sugar = 400 - 240 = 160 g.

15. An inheritance is shared between three children. The eldest receives 50% of the total, the middle child receives 40% of what the eldest gets, and the youngest receives the remaining £24,000. Find the total inheritance and the ratio of their shares in the form *Eldest : Middle : Youngest*.

    Solution:
    Let total = T. Eldest = 0.5T. Middle = $0.4 \times (0.5\text{T}) = 0.2\text{T}$.
    Youngest = $T - (0.5T + 0.2T) = 0.3T = 24000$.
    So $T = \frac{24000}{0.3} = 80,000$.
    Eldest = 0.5×80000 = £40,000. Middle = 0.2×80000 = £16,000. Youngest = £24,000.
    Ratio = 40000 : 16000 : 24000 = 40 : 16 : 24 = 5 : 2 : 3 (dividing by 8).

Sharing Into Ratios Future

16. The perimeter of a rectangle is 30 cm. The sides are in the ratio 2:3. Find the area of the rectangle.

    Solution:
    Let sides be $2x$ and $3x$. Perimeter = $2(2x + 3x) = 10x = 30 \implies x = 3$.
    Sides: 6 cm and 9 cm. Area = $6 \times 9 = 54$ cm^2.

17. The area of a rectangle is 54 cm^2. The sides are in the ratio 2:3. Find the perimeter of the rectangle.

    Solution:
    Let sides be $2x$ and $3x$. Area = $2x \times 3x = 6x^2 = 54 \implies x^2 = 9 \implies x = 3$.
    Sides: 6 cm and 9 cm. Perimeter = $2(6 + 9) = 30$ cm.

18. A right-angled triangle has sides in the ratio 3:4:5. The area of the triangle is 96 cm^2. Find the perimeter of the triangle.

    Solution:
    Let sides be $3x, 4x, 5x$. The two shorter sides are perpendicular.
    Area = $\frac{1}{2} \times 3x \times 4x = 6x^2 = 96 \implies x^2 = 16 \implies x = 4$.
    Sides: 12 cm, 16 cm, 20 cm. Perimeter = $12 + 16 + 20 = 48$ cm.

19. A garden is in the shape of an L-shape made from two rectangles. The first rectangle is 3m by 2m. The second rectangle has sides in the ratio 3:4, If the total area of the garden is 54 m^2, find the dimensions of the second rectangle.

    Solution:
    Area of first rectangle $= 3 \times 2 = 6$ m^2.
    Area of second rectangle $= 54 - 6 = 48$ m^2.
    Let sides of second rectangle be $3y$ and $4y$.
    Area $= 3y \times 4y = 12y^2 = 48 \implies y^2 = 4 \implies y = 2$.
    Dimensions: $3 \times 2 = 6$ m and $4 \times 2 = 8$ m.

20. A circular pond is surrounded by a path. The radius of the pond and the width of the path are in the ratio 2:1. If the total area (pond + path) is 99π m^2, find the width of the path.

    Solution:
    Let pond radius $= 2x$, path width $= x$. Then outer radius $= 2x + x = 3x$.
    Total area $= \pi(3x)^2 = 9\pi x^2 = 99\pi \implies x^2 = 11 \implies x = \sqrt{11} \approx 3.32$ m.
    Width of path $= \sqrt{11}$ m.

Ratio Problem Solving Past

21. In a class, the ratio of boys to girls is 3 : 2. 40% of the boys wear glasses. 20% of the girls wear glasses, what percentage of the *whole class* wear glasses?

    Solution:
    Let number of boys $= 3k$, girls $= 2k$, total $= 5k$.
    Boys with glasses $= 0.4 \times 3k = 1.2k$. Girls with glasses $= 0.2 \times 2k = 0.4k$.
    Total with glasses $= 1.2k + 0.4k = 1.6k$.
    Percentage $= \frac{1.6k}{5k} \times 100\% = 32\%$.

22. The price of a rare coin is increased by 20%. The new ratio of its price to its original price is $a : b$. Write the ratio $a : b$ in its simplest form.

    Solution:
    Let original price $= P$. New price $= 1.2P$.
    Ratio new : original $= 1.2P : P = 1.2 : 1 = 12 : 10 = 6 : 5$.

23. A shop sells only two types of fruit: apples and oranges. The ratio of apples to oranges is 5 : 3. 30% of the apples are green, and the rest are red. 60% of the oranges are from Spain, and the rest are from Morocco. What is the ratio of **green apples** to **oranges from Morocco**?

    Solution:
    Let apples $= 5k$, oranges $= 3k$.
    Green apples $= 0.3 \times 5k = 1.5k$.
    Oranges from Morocco $= (1 - 0.6) \times 3k = 0.4 \times 3k = 1.2k$.
    Ratio $= 1.5k : 1.2k = 15 : 12 = 5 : 4$.

24. In a library, the ratio of fiction to non-fiction books is 7 : 3. After a donation, the number of fiction books increased by 15% and the number of non-fiction books

decreased by 10%. Find the new ratio of fiction to non-fiction books. Give your answer in the form $A : B$ where A and B are integers.

Solution:
Let fiction $= 7k$, non-fiction $= 3k$.
New fiction $= 7k \times 1.15 = 8.05k$.